Numerical Modelling of Sediment Transport and Consolidation in the Aswan High Dam Reservoir

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1. Abstract

Engineers involved in river regulation and design of water reservoirs have a great need for methods of computing sediment discharge and sediment consolidation. In this work a numerical model for sediment transport and consolidation in the Aswan High Dam Reservoir was designed and used for this purpose. The model is based on the principle equations of water volume conservation, water momentum conservation and a general sediment transport equation based on an improved rating curve for Dongola station at the inlet and a modified shields' equation at the reservoir. Also, an improved consolidation model independent of the sedimentation time was used, where the average bed density is calculated as a mass balance of the existing consolidating sediments influenced by deposition or erosion. The model gives good results compared to the actual measurements in the period May 1964 to November 1988 and consequently provides reliable predictions of sediment transport and consolidation.

Keywords

Sediment transport, consolidation, Rating curve, Water volume conservation, water momentum conservation.

2. Introduction

Prior to the construction and operation of High Aswan Dam, in 1964, 9-10 Million tons of suspended sediment were deposited annually in the flood plain of the Nile, while about 93% of the total average annual suspended load of 124 Million tons was carried out into the Mediterranean Sea. Since the full operation of the HAD in 1968, the flood discharge of the Nile, below the dam, has been greatly modified and more than 98% of the total suspended load has been retained within the reservoir and the total amount of silt transported below the reservoir dropped to 2.5 million tons per year (Fig.1).

3. Significance of the Problem and the Aim

Year by year the sediment is moving close to the dam and there are simultaneous moveable waves of erosion and deposition. The erosion may affect the stability of the dam and the deposition will affect the storage capacity of the reservoir. Therefore it is important to predict the bed profile of the reservoir in future. The objective of this paper is the formulation of a numerical model for unsteady state surface water flow and sediment movement in time to simulate sediment transport in the AHD reservoir. The approach discussed herein uses the general finite difference method. Of particular importance is the development of a FORTRAN 77 computer program using a unified code representation, a structured input data file that can be easily understood by the user.

4. Analysis of the available data

For any numerical model of sediment transport, the input data for sediment load and discharges are essential. There is no continuous record of suspended sediment concentrations; therefore it was proposed to use the rating curves.

4.1 The Dongola rating curve

As indicated in Bulletin 110 HADSERI (El-Moattassem and Makary, 1988), the Dongola rating relationship was developed from the total set of available observations. The mathematical equation of this curve

$$q_s = 2.926 \times 10^{-8} q^{2.788} \qquad (1)$$

where q_s is the sediment load in 10^9 kg/d
q is the discharge in 10^6 m^3/d

This equation has been used to estimate the deposited sediment in the period 1964 - 1985, the estimated deposited sediment was 2.185×10^{12} Kg. Applying Lane formula (1953) and Miller formula (1953) for calculating the density of the deposited sediments, the estimated deposited sediment corresponded to a volume of 1.65×10^9 m^3. The volume of deposited sediments was calculated also from the hydrographic survey in the same period and it was 1.646×10^9 m^3, therefore this rating curve was concluded to be accurate.

Remarks on the Dongola rating curve

1. The rating curve has been derived by simple regression of the sediment load as a function of the discharge measurements. Flow velocities were measured to determine discharges. At each cross section in Aswan High Dam reservoir the velocity profile and direction of flow were measured in three places only, while they should be measured in twenty places at least to be accurate. This applies to large as well as to small rivers (Jansen 1979). This indicates that large errors are present for discharge while the suspended sediment concentrations can be measured accurately. Therefore it will be more logical to invert the regression, that is, the regression has to be done for the discharge as a function of the sediment load measurements.

2. In the estimation of the deposited sediment in the period 1964 -1985 from the hydrographic survey, the last four cross sections, from km 132 to km 175, were neglected. These four cross sections are 10 times larger than the cross sections in the inlet zone, i.e. the largest amount of deposited sediment in the reservoir was neglected. Consequently, the estimation of the deposited sediment from the hydrographic survey is not correct, and the rating curve is also inaccurate.

4.2 An improved Dongola sediment rating curve

We assume a relation of the following form

$$Q_s = a + b.(Q)^c \quad \text{or} \quad S.Q = a + b.(Q)^c \qquad (2)$$

Where S: is the average sediment concentration in (Kg/m^3)
Q_s: is the sediment load in (Kg/s)
Q: is the discharge in (m^3/s)
a, b and c are constants.

Therefore $S = a + b.(Q)^{c-1}$ (3)

$$\frac{S - a}{b} = Q^{c-1} \qquad (4)$$

or

$$Q = \left(\frac{S - a}{b}\right)^{\frac{1}{c-1}} \quad (5)$$

S can be measured accurately but large errors are present for Q, hence a, b and c should be determined such that the sum of squared differences between measured and calculated Q values is minimum. This can be done with non-linear optimization programs. Based on the least square difference, the three coefficients a, b and c were calculated and the mathematical equation of the improved rating curve was found:

$$Q_s = 0.1\ Q + 8.9\text{x}10^{-11}\ Q^{3.744} \quad (6)$$

This relation is shown in Figure 2.

4.3 The reservoir rating curve

The available data sets of November 1986 and November 1988 were used to calculate the sediment load per unit width Q_b as a function of velocity. Since the sediment load can be measured accurately, it was more logical to do the regression of the velocity as a function of the sediment load; and the following equation was found

$$Q_b = 1.2 \text{ x } U \quad (7)$$

where U: is the velocity of the flow in (m/s)
Q_b: is the sediment load per unit width (Kg/m.s)

In the measurements of November 1986 and November 1988, the higher velocities were missing; therefore it was necessary to modify this equation to have a good representation either to low or high velocities. Therefore theoretical models have been tested using the available data of the sedimentation zone, which starts from the inlet section at Km 12.5 to Km 175. The tested models are:

(1) Bagnold's model.
(2) Meyer-peter and Muller Model.
(3) Shields model.

In view of the results, it is noted that Bagnold's model could be applied only in the inlet zone and the estimated load per unit width is close to the measured one when the velocity is high. Meyer-peter and Muller model does not give reliable results. Shields gives good results almost along the whole reservoir especially in case of high discharge. Based on these results, it was proposed to use a shields equation for the rating curve in the reservoir

4.4 Shields equation

$$Q_b = 10\, q_0\, i \frac{\tau_0 - \tau_c}{(\frac{\rho_s}{\rho} - 1)\ d_{50}} \quad (8)$$

Where Q_b: is the sediment load per unit width in (kg/m.s)
q_0: is the water discharge per unit width in (m^3/m.s)
i: is the slope of the energy gradient line
d_{50}: is the mean diameter of bed material in (m)
τ_0: is the average shear stress in (N/m^2)
τ_c: is the critical shear stress in (N/m^2)
ρ: is the density of fluid in (Kg/m^3)
ρ_s: is the density of sand in (Kg/m^3)

$$q_0 = \frac{Q}{A} = \frac{AU}{B} = h.U \quad (9) \qquad U = c\sqrt{h * i}$$

where h: hydraulic radius in (m)
c: Chezy coeff. = 50 $m^{0.5}$/s
g: is the gravity acceleration in (m/s^2)

$$i = \frac{U^2}{c^2 h} \quad (10) \qquad \tau_0 = \rho g (\frac{U}{c})^2$$

From the actual measurements in the Aswan High Dam reservoir during the period 1964 - 1988, it was found that the mean diameter of bed material= 0.14 m, and the average critical shear stress= 0.02 Newton/m^2.
Then

$$Q_b = 10 \frac{U^3}{g(50)^2} \left(\frac{1000(9.81)\frac{U^2}{(50)^2} - 0.02g}{2.7225\, d_{50}} \right) \quad (11)$$

$Q_b = 4.16\ U^5 - 0.21\ U^3$ (12)

A comparison between the sediment load per unit width Q_b measurements in November 1986, November 1988 and the calculated values using Shields equation has been made. It is noted that Shields curve is accurate only for the high velocities; therefore the following equation was proposed:

$$Q_b = 1.2\ U + 4.16\ U^5 - 0.21\ U^3 \qquad (13)$$

This model rating curve fits the measurements and it is tangent to Shields curve in the high measurements, therefore the model rating curve is more appropriate, as shown in Figure 3.

5. Consolidation theory

The theoretical work for consolidation has not progressed to the point at which it can be used to predict densities of reservoir deposits. The empirical relations such as Lane and Koelzer formula (1953) and Miller formula (1953) give the deposits in reservoirs at the end of T years or the average density of the sediments in a reservoir after T years of operation during which deposits accumulated at a uniform rate. In the model the density of sediments is needed in every time step but the rate of deposition is not uniform. Therefore a more convenient method for the consolidation of the sediments will be used. The basic idea of this method is that the relation between the density deposit and time is assumed to follow an exponential curve as shown in figure 4.
The density is a function of time in an exponential relation

$$\rho = \rho_{min} + (\rho_{max} - \rho_{min})(1 - e^{-t/t_c}) \qquad (14)$$

Where t_c: is the characteristic consolidation time,
e : is the natural log base
t : is time

ρ_{min} : the initial density at the time of sedimentation or the start of the consolidation

ρ_{max} : the maximum density of the accumulated sediments at the end of the consolidation.

Hence, the consolidation rate is given by:

$$\frac{d\rho}{dt} = \frac{1}{t_c}(\rho_{max} - \rho_{min})e^{-t/t_c} = \frac{\rho_{max} - \rho}{t_c} \qquad (15)$$

or

$$\frac{dZ_b}{dt} = -\frac{1}{t_c}(Z_b - Z_{b_0})(\frac{\rho_{\max}}{\rho}-1) \tag{16}$$

Where Zb: is the bed level at any time step

Z_{b0}: is the initial bed level

Hence, at all times the change in bed level due to consolidation can be calculated if the density is known.

6. Model Equations

Water Volume Conservation

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \tag{17}$$

Where A: is the cross sectional area = h.b

h: is the water depth

b: is the width of the cross section at the water surface

Q: is the discharge

q_l: is the lateral inflow or outflow

1. Since the flow is gradually varied, i.e. The Froude Number values are moderate; we can approximate the flow from the Non Steady flow to Pseudo flow.

2. For more or less prismatic channels the change of the cross sectional area with respect to distance is almost equal.

3. For wide cross sections, the hydraulic radius equals the depth of the flow.

There is no lateral inflow or outflow in the reach between Dongola station and the inlet of the sedimentation zone, therefore we will assume $q_l = 0$

$$b\frac{\partial h}{\partial t} + h\frac{\partial b}{\partial t} = -\frac{\partial Q}{\partial x} \tag{18}$$

$$(b + h\frac{\partial b}{\partial h})\frac{\partial h}{\partial t} = -\frac{\partial Q}{\partial x} \tag{19}$$

$$h = Z_w - Z_b \dashrightarrow \frac{\partial h}{\partial t} = \frac{\partial Z_w}{\partial t} - \frac{\partial Z_b}{\partial t} \tag{20}$$

where Z_w: is the water level
Z_b: is the bed level

Since

$$\frac{\partial Z_b}{\partial t} \prec\prec \frac{\partial Z_w}{\partial t}$$

Then equation 20 tends to

$$\frac{\partial h}{\partial t} = \frac{\partial Z_w}{\partial t} \tag{21}$$

Hence,

$$(b + h\frac{\partial b}{\partial h})\frac{\partial Z_w}{\partial t} = -\frac{\partial Q}{\partial x}$$

Water Momentum Conservation

$$Q = (-\frac{\partial Z_w}{\partial x})Ac\sqrt{h\,|\frac{\partial Z_w}{\partial x}|} \tag{22}$$

Sediment Transport

For quasi-steady state, one dimensional flow:

$$B(1 - p)\rho\frac{\partial Z_b}{\partial t} + \frac{\partial Q_s}{\partial x} = 0 \tag{23}$$

B: is the width of the cross section at the bottom level
Z_b: is the bed level elevation
p: is the sediment porosity
ρ: is the density of sediment particle (2650 Kg/m^3)
Q_s: is the sediment load

Let ρ_d = (1- p) ρ_s
= sediment dry density which is time dependent because of consolidation. Hence,

$$B \rho_d \frac{\partial Z_b}{\partial t} = - \frac{\partial Q_s}{\partial x} \tag{24}$$

In order to calculate Q_b, we will use an empirical relation of the form
Qs = Q_b(U)*B where U = Q/A the average flow velocity.
Sediment load at the whole reservoir at the time step t is given by the reservoir rating curve.

Boundary conditions are only needed at the inlet where the sediment load is given by the Dongola rating curve.

Sediment Consolidation Equations

At all times the change in bed level due to consolidation can be calculated if the density is known from the relation (16).

$$\frac{\partial Z_b}{\partial t} = \frac{1}{t_c}(Z_b - Z_{b_0})(\frac{\rho_{\max}}{\rho} - 1)$$

Where t_c: = 14 years
ρ_{max}: is the maximum density of the accumulated sediments at the end of the consolidation time (one of the model parameters = 1200 Kg/m^3)

Sedimentation

$$\frac{\partial (Z_b - Zb0)}{\partial t} = \frac{\rho}{\rho \min}$$

ρ_{min}: is the minimum density (one of the model parameters = 1000 Kg/m^3)

$$\frac{d\rho}{dt} = \frac{r(1 - \rho / \rho_{\min})}{Z_b - Z_{b_0}} \tag{25}$$

Erosion

$$\frac{d(Z_b - Z_{b_0})}{dt} = \frac{r}{\rho} --- \rightarrow \frac{d\rho}{dt} = 0 \tag{26}$$

7. Description of the Model

Sediment transport and consolidation problems have been described by the computer program SEDIMENT MODEL. The program solves the partial differential equations governing the water flow, sediment transport and consolidation under Pseudo state conditions, using equations number 4, 13, 16, 22, 23, 28 and 29. For the water movement a simple technique has been used to increase the time step. In this technique monthly average discharges have been used with the assumptions that the discharge does not depend upon distance because there is no lateral inflow or outflow in the area of study and water level in every month is a function of distance only.

The input data necessary for this program are described in the data file INPUT.DAT which contains the number of the cross sections and their distances from the inlet, the minimum bed level in 1964, width as a function of water level at every cross section, the input discharge as measured at Dongola station as the inlet discharge and fixed water levels at the outlet section equal to the mean annual water level measurements.

The output results are given in the file OUTPUT.DAT. The output results are the calculated average bed level, the calculated water level, water depth, width of the cross section, cross sectional area, velocity, sediment load, sediment load per unit width, the density and the calculated suspended sediment concentration at every cross section at definite date. Also the calculated input total sediment load up to that date, the calculated output total sediment load and the calculated deposited sediment at the whole sedimentation zone (from Km 0 to Km 175).

8.1 The results of the bed shape

It is observed that the calculated bed level is somewhat higher than the measured in the inlet zone, from Km 12.5 to Km 122 (Figure 5). In the model however, the cross sections were considered to be rectangular with the same width at the water level and an average depth. This means that the measured bed level is the lowest while the calculated is an average. This may explain that difference. In the outlet zone, the difference between the measured and model cross sections is small. Generally, the comparison between the measured and the estimated bed levels indicates good results.

8.2 The results of November 1988

The model and the measured backwater curve is almost the same along the whole sedimentation zone except at the first two cross sections in the inlet where small differences are observed (Figure 6).

There is a similar trend between the calculated and the measured velocities, sediment load and suspended sediment concentrations. The calculated and measured parameters are very close in the outlet zone and the inlet cross section while there is a considerable difference in the middle zone.

The measured discharges have been used as inflow in the input data file along the whole sedimentation zone in the model. This implies that there is no difference between the measured and the model discharges. As the velocity is a function of the discharge and the cross sectional area, the difference between the calculated and the measured velocities were referred to the difference between the model and the measured cross sections.

8.3 Comparison of calculated and measured sediment volumes

The model result for the total mass of the accumulated sediment during the period May 1964 to November 1988 is 3329×10^9 kg. The result of the density of the sediment in November 1988 along the whole sedimentation zone ranges between 1079 Kg/m^3 and 1163 Kg/m^3 with an average 1121 Kg/m^3. This indicates that the model result for the total volume of the deposited sediment is $2.97 \times 10^9 m^3$ including the effect of sediment consolidation. From the hydrographic survey, the estimation of the deposited sediment volume in the same period has been done and the result is $2.995 \times 10^9 m^3$. This implies that the model results for deposited sediment are very close to the measured values, considering the consolidation phenomena

8.4 Future predictions

One of the main aims of this research is to predict the future evolution of the parameters studied (Fig.7). We assume that the discharges in the period 1964 to 1988 will occur in the same way in the next 25 years. Starting from the water level in 1988, we assume that there will be a slight increase in the water level until the reservoir will be full, and this increasing period will be followed by a decreasing period. This assumption is similar to the measurements of the mean annual water level in the period 1964 to 1988.

Our forecast shows that the deposition will continue in the same way and the bed level will reach 173 m by the year 2000 in the whole inlet zone until Km 140. This deposition will be followed by an erosion period in the same zone until the bed level reach 162 m by the year 2010. Consequently the eroded sediment will move closer to the dam and will be deposited up to Km 170.

Conclusions

Based on the results for the different parameters involved in sediment transport and consolidation processes, we conclude that:

1. The estimated bed shape and backwater curves are very close to the field measurements along the whole sedimentation zone.

2. The predicted accumulated deposited sediment taking into consideration the consolidation phenomena during the period May 1964 to November 1988 is very close to that of measured using the hydrographic survey.

3. The results indicate that the model is very close to natural reservoir conditions and allows a comprehensive and physical sound interpretation of sediment transport and consolidation.

Recommendations for future research

1. The results can be improved by using a two-dimensional model to consider the effect of the bed forms in the transverse direction.

2. The effect of the sediment constituents on the sediment transport could not be evaluated properly using the model facilities. Further research in this area is necessary.

Concerning the accuracy of the field measurements

In order to assess the adequacy of the results obtained by the model it is recommended that:

1. The velocities and the suspended sediment concentrations should be measured at least in 20 vertical locations instead of 3 in every cross section.

2. The field investigation should be carried out two or three times per year instead of one, especially during the flood period, in order to improve the whole reservoir sediment rating curve.

3. The measurements should be taken in the inlet (cross section 23) instead of Dongola station which is 250 km from the area of study.

4. It is necessary to take the measurements at El Gaafra station (34 km downstream of the dam) beside the measurements at the outlet cross section of the sedimentation zone.

REFERENCES

Bear, J and A. Vrruijt, 1987," Modeling Groundwater and Pollution with Computer Programs for Sample Cases", D. Reidel Publishing Company Dordrecht.

Chen Y. A., P. F. Lagasse, S. A. Shumm, D. B. Simons,1966," The River Environment", Colorado State University, Fort Collins, USA.

ElMoattassem, M. and T.M. Abdel-Aziz, 1988," A Study of the Characteristics of Sediment Transport in the Aswan High Dam Reservoir", Bulletin 117 Aswan High Dam Side Effects Research Institute, Cairo, Egypt.

ElMoattassem, M. and A. Z. Makary, 1988," Sedimentation Balance in the Aswan High Dam Reservoir", Bulletin 110 Aswan High Dam Side Effects Research Institute, Cairo, Egypt.

Graf, W. and C. H. Mortimer, 1979," Developments in Water Science Hydrodynamics of Lakes", Elsevier, Amsterdam.

Graf, W., 1971," Hydraulics of Sediment Transport", McGraw-Hill, New York, USA.

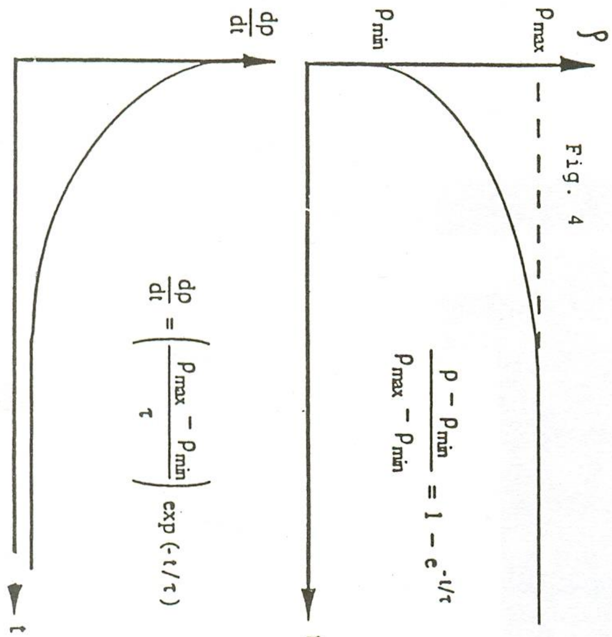
Jansen, P. Ph., L Van Bondegom, J Van Bondegom, J Van den Berg, M de Vreis and A Zanen, 1979," Principles of River Engineering: The Non-Tidal Alluvial River", Pitman, London.

Kobus, H. and G. Abrahame, 1980," Hydraulic Modelling", Pary, Hamburg.

Lang, S., 1979," Calculus of Several Variables", Addison Wesley, London.

Des E. Walling, A. Yair and S. Berkowicz, 1990," Erosion, Transport and Deposition Processes", IAHS, Publication; 189, Oxfordshire, U.K.

Dole W. and G. Guariso, 1983," Water Management Models in Practice: A case Study of the Aswan High Dam", ELsevier, Amsterdam.



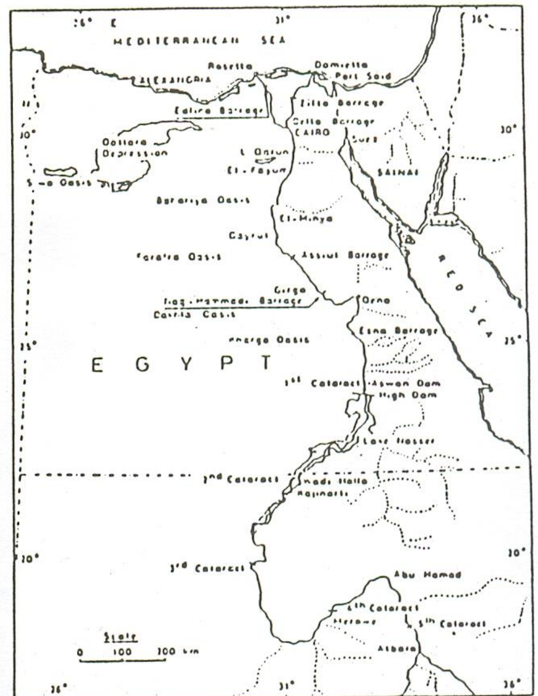
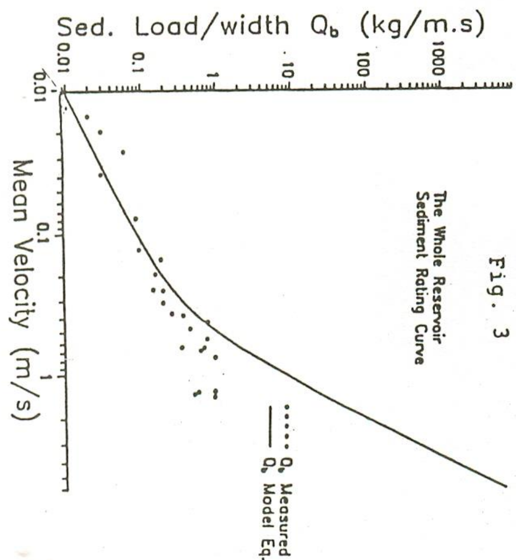


Fig. 1 Map showing the location of the Nile control works in Egy
Mamdouh Shahin, 1985, Hydrology of the Nile Basin, p. 448.



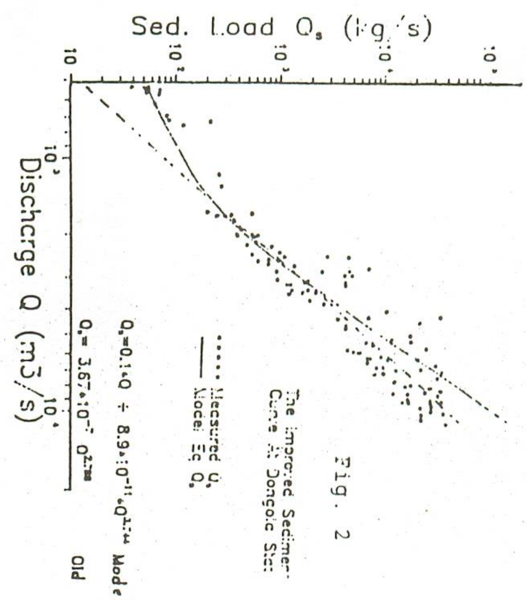


Fig. 5, BED SHAPE

Model & Measured Longitudinal Sections

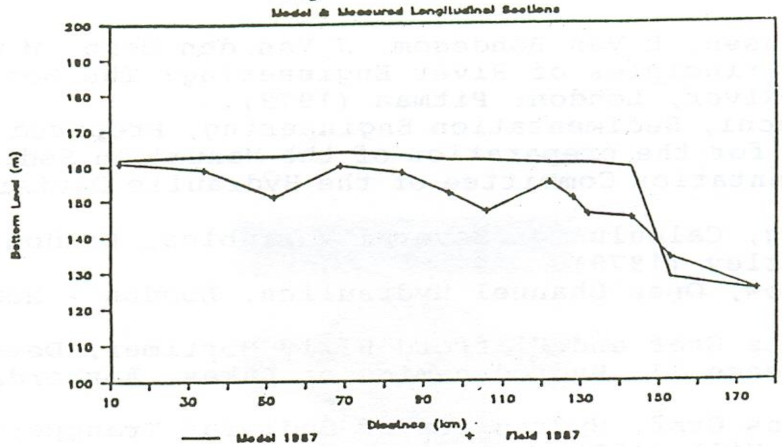


Fig. 6 Comp.bet. Model & Field

Back Water Curve (November 1988)

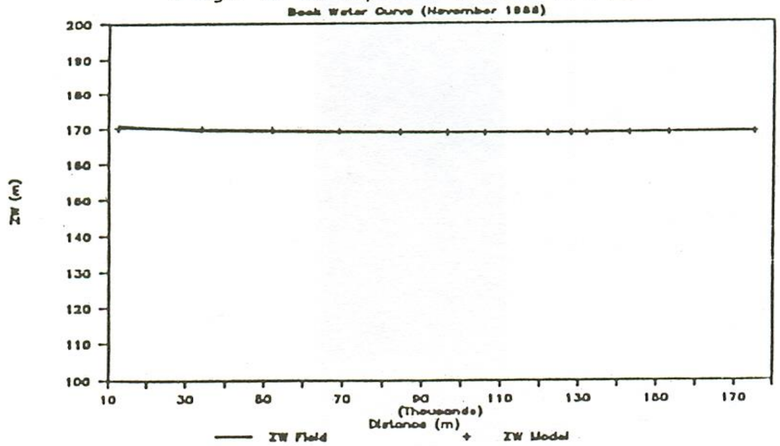


Fig. 7.) Future Predictions

Bed Shape (Nov.2000 & Nov.2010)